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Probability & Statistics

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Problem

x	0	1	2	3	4	5	6	7	8
y	12	10.5	10	8	7	8	7.5	8.5	9

For the above data,

(a) Find the regression line

(b) Find 95% confidence interval for $\mu_{Y|_{x=2}}$.

(c) Find 99% confidence interval for β_1

(d) Find 90% confidence interval for β_0

(e) Find 96% confidence interval for σ^2.

Problem

(f) Test the hypothesis $H_0 : \beta_1 = 0$ against

$H_1 : \beta_1 < 0$ at 5% level.

(g) Find Type-II error for $\beta_1 = -0.1$.

(h) Test the hypothesis: $\beta_0 = 7$ at 1% level.

Theorem

(A) $\mathrm{Var}\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \mathrm{Var}(X_i)$

(B) $\mathrm{Cov}\left(\sum_{i=1}^{n} a_i X_i \,,\, \sum_{j=1}^{m} b_j Y_j\right) = \sum_{i=1}^{n}\sum_{j=1}^{m} a_i b_j \mathrm{Cov}(X_i, Y_j).$

Theorem: $\text{Cov}\left(\bar{Y}\ ,\ B_1\right) = 0.$

Proof: Since $\bar{Y} = \sum_{i=1}^{n} (1/n) Y_i$ and $B_1 = \sum_{j=1}^{n} \left(\frac{x_j - \bar{x}}{S_{xx}} \right) Y_j$

$$\Rightarrow \text{Cov}(\bar{Y}, B_1) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{x_j - \bar{x}}{nS_{xx}} \right) \text{Cov}(Y_i, Y_j)$$

As Y_i 's are independent which implies $\text{Cov}(Y_i, Y_j) = 0$ for $i \neq j$.

$$\therefore\ \text{Cov}(\bar{Y}, B_1) = \left(\frac{1}{nS_{xx}} \right) \sum_{i=1}^{n} (x_j - \bar{x}) \text{Cov}(Y_i, Y_i)$$

$$= \left(\frac{1}{nS_{xx}} \right) \sum_{i=1}^{n} (x_j - \bar{x}) \text{Var}(Y_i) = \left(\frac{\sigma^2}{nS_{xx}} \right) \sum_{i=1}^{n} (x_j - \bar{x}) = 0.$$